**Smart Home Security System: Boolean Algebra and Logic Gates**

**Question** 1:

1. **Boolean experssion**
2. Motion sensor in the living room is triggered:

Let M = Motion sensor triggered

Expression: M

ii. Front door is opened:

Let D = Front door is opened

Expression: D

**b. Boolean algebra: M and D are already single variables representing single conditions, they are in their simplest form.**

**Question** 2:

a. Apply De Morgan's Theorems:

i. Smoke is detected in the kitchen:

Let S = Smoke detected in kitchen

Negation: NOT (S) → S'

Applying De Morgan's Theorem isn't necessary for a single variable, but if it were part of large experession like "Smoke detected AND CarbonMonoxide detected (C)," the negation would be:

Not (S AND C) = S' OR C'

ii. Windows are closed:

Let W = Windows are closed

Negation: NOT (W) → W'

Since it’s a single condition, De Morgan's Theorem mainly applies when multiple conditions are involved.

**b. Relationship between De Morgan's Theorems and other Boolean algebra laws:**

De Morgan's Theorems describe how to distribute a NOT across AND/OR operations:

(A AND B)' = A' OR B'

(A OR B)' = A' AND B'

This is related to the complementarity laws and the distributive laws in Boolean algebra, they help in simplifying complex expressions when dealing with negated conditions.

**Question 3:**

**a. Truth Table for "Motion sensor in the living room" (AND Gate):**



**b. Truth Table for "Security alarm armed NOR Windows closed" (NOR Gate):**



**Question 4:**

**a. Equivalence between Boolean expressions and Truth Tables:**

* The Boolean expression M AND D matches the truth table for the AND gate, where output is 1 only if both M and D are 1.
* The NOR truth table for A NOR W corresponds to the expression (A + W)', where output is 1 only when both A and W are 0.

b. Algebraic manipulations and logical reasoning:

* AND operation:

M AND D = M \* D

Matches output 1 only when both inputs are 1.

* NOR operation:

NOR means complement of OR: (A + W)'

Follows De Morgan’s law: NOT (A OR W) = A' AND W'

Thus, the simplified Boolean expressions are perfectly reflected in their truth tables.

**Question 5:**

a. Integrate simplified Boolean expressions: Overall logic

* Condition 1: If the motion sensor AND door sensor are both active Intrusion detected.
* Logic: M AND D
* Condition 2: If the alarm is NOT armed AND the windows are closed Normal, no alert.
* Logic: (A + W)' (using NOR gate)

**Combined System Behaviour:**

* If M AND D = 1--- Trigger Intruder Alert.
* If (A + W)' = 1 --- System remains in secure state.
* Otherwise, depending on specific triggers (e.g., smoke, other sensors), additional actions could be layered.

**Diagram of simple logic overview:**

[M]----\

AND ----> [Intruder Alert]

[D]----/

[A]----\

OR

[W]----/

NOT ----> [System Secure]

Applying Boolean algebra, De Morgan’s Theorems, and truth tables, we successfully designed a logic-based smart home security system.

**References:**

Mano, M. M., & Ciletti, M. D. (2017). Digital design (6th ed.). Pearson.

UoPeople Learning Resource Center. (2025). Boolean Algebra and Logic Gates Materials.